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PP wave limit and enhanced supersymmetry in gauge theories

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Abstract: We observe that the pp wave limit of $AdS_5 \times M^5$ compactifications of type-IIB string theory is universal, and maximally supersymmetric, as long as M^5 is smooth and preserves some supersymmetry. We investigate a specific case, $M^5 = \mathbb{T}^{1,1}$. The dual $\mathcal{N} = 1$ SCFT, describing D3-branes at a conifold singularity, has operators that we identify with the oscillators of the light-cone string in the universal pp wave background. The correspondence is remarkable in that it relies on the exact spectrum of anomalous dimensions in this CFT, along with the existence of certain exceptional series of operators whose dimensions are protected only in the limit of large 't Hooft coupling. We also briefly examine the singular case $M^5 = S^5/\mathbb{Z}_2$, for which the pp wave background becomes a $\mathbb{Z}_2$ orbifold of the maximally supersymmetric background by reflection of 4 transverse coordinates. We find operators in the corresponding $\mathcal{N} = 2$ SCFT with the right properties to describe both the untwisted and the twisted sectors of the closed string.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua, AdS/CFT Correspondence, Extended Supersymmetry.

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Contents

1. Introduction

According to the AdS/CFT conjecture [1, 2, 3], the chiral operators of the $\mathcal{N}=4$ supersymmetric SU(N) gauge theory are in one-to-one correspondence with the modes of type-IIB supergravity on $AdS_5 \times S^5$. The massive string modes, however, correspond to operators in long multiplets whose dimensions diverge for large 't Hooft coupling as $(g^2_{\rm YM}N)^{1/4}$. For this reason, the precise map between massive string modes and gauge invariant operators has been difficult to construct at strong coupling. Recently, however, major progress in this direction has been made by Berenstein, Maldacena and Nastase (BMN) [4]. Their proposal is to consider states with very large angular momentum along the great circle of S^5, $J \sim \sqrt{N}$. The metric felt by such states is the Penrose limit of $AdS_5 \times S^5$, which is [5, 6] the pp wave

$$ds^2 = -4dx^+dx^- + \sum_{i=1}^{8}(dx_i)^2 - \mu^2(dx^+)^2 \sum_{i=1}^{8} x_i^2 \tag{1.1}$$

supported by the 5-form RR field strength

$$F_{+1234} = F_{+5678} \sim \mu\,. \tag{1.2}$$

The 5-form breaks the SO(8) symmetry of the metric down to SO(4) $\times$ SO(4). The pp wave limit preserves 32 supercharges, as many as the $AdS_5 \times S^5$ background [5, 6].

This string background is remarkable in that the string theory is exactly sovable in spite of the presence of the RR 5-form field strength. As shown by Metsaev [7] (see also [8]), in the light-cone gauge the 8 world sheet fields describing the transverse coordinates, and their fermionic superpartners, all acquire the same mass μ. Therefore, the light-cone hamiltonian for a single string assumes the form

$$p^- = \sum_{n=-\infty}^{\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}}\,, \tag{1.3}$$

where N_n is the excitation number of the n-th oscillator.

BMN combined this formula with the AdS/CFT duality to construct the following relation between the dimension Δ and the R-charge J of the corresponding gauge theory operator [4]:

$$\Delta - J = \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{4\pi g_s N n^2}{J^2}} \,. \tag{1.4}$$

This formula is valid for $\frac{\Delta - J}{J} \ll 1$. In an ingenious construction, BMN identified the set of single-trace operators which they argued have these dimensions and R-charges. They suggested that this class of single-trace operators is in one-to-one correspondence with the single string spectrum in the pp wave background (1.1).

This remarkable result raises many interesting questions. The questions we would like to ask are: is the maximally supersymmetric gauge theory necessary for reconstructing the full string spectrum? Can an analogous construction be carried out with a gauge theory that has reduced supersymmetry or no supersymmetry at all? In fact, in [4] it was pointed out that the Penrose limit of the $\mathcal{N} = 2$ supersymmetric orbifold $AdS_5 \times S^5/\mathbb{Z}_2$ is a $\mathbb{Z}_2$ orbifold of the pp wave under $x_i \to -x_i$, $i = 5, 6, 7, 8$. In section 4 we consider the matching of string states in this background with the single-trace operators of the $\mathcal{N} = 2$ $\mathrm{SU}(N) \times \mathrm{SU}(N)$ orbifold gauge theory [11].

Furthermore, we study the Penrose limits of spaces $AdS_5 \times \mathbb{T}^{p,q}$ which preserve $\mathcal{N} = 1$ superconformal symmetry for $p = q$. We will primarily consider the basic case $p = q = 1$ which is dual to an $\mathcal{N} = 1$ superconformal $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory [12]. For $p = q > 1$ we find a $\mathbb{Z}_p$ orbifold of this gauge theory: an $\mathcal{N} = 1$ superconformal $\mathrm{SU}(N)^{2p}$ gauge theory which we will not discuss here in any detail. For $p \neq q$ the supergravity background is non supersymmetric and is, actually, unstable [17]. Remarkably, for $p = q$ the Penrose limit is the maximally supersymmetric pp wave background (1.1).[1] This is rather puzzling since the gauge theory of [12] has only 1/4 of the maximal supersymmetry. The only logical possibility seems to be that the operators surviving in the appropriate limit of large R-charge form a subsector with enhanced supersymmetry. While this is difficult to prove, we will provide evidence that this is indeed the case.

Our construction of the string states largely parallels the BMN construction. It will be very important for us that in addition to the $\mathrm{U}(1)_R$ symmetry the gauge theory possesses $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry. The correct quantum number for classifying the string states (the analogue of $\Delta - J$ of [4]) turns out to be

$$H = \Delta - \frac{J}{2} + J_3 + J_3' \,, \tag{1.5}$$

where J is the R-charge while J_3 and J_3' are the $\mathrm{SU}(2)$ and $\mathrm{SU}(2)'$ quantum numbers. We find a family of $H = 0$ operators: $\mathrm{Tr}(A_2 B_2)^J$.[2] The insertion operators corresponding to the 8 transverse bosons and fermions carry $H = 1$ and are presented in section 3.

[1]For $p \neq q$ we find a more general pp wave metric which gives, in the light-cone gauge, different masses for the oscillators in the 1234, 56 and 78 directions.

[2]There also exist dibaryon operators [13] carrying $H = 0$: $\det A_2$ and $\det B_2$. These operators are of no concern to our paper since they carry $J = N$, much greater than in the BMN scaling for closed strings. These operators instead correspond to wrapped D3-branes [13]. Such operators and their excitations may therefore be relevant to generalization of the BMN construction to D-branes and open strings [14].

JHEP03(2002)048

It is interesting that some of the operators we have to use are not chiral. For $J = 0$ they have protected dimensions due to the presence of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry. For $J > 0$ it is likely that the dimensions are not protected. However, we are able to use the AdS/CFT correspondence to show that these operators have $H = 1$ in the limit of very large $g_s N$. This is in accord with the expectation that there is a subsector of the gauge theory that has enhanced symmetry in the appropriate limit of large quantum numbers and large $g_s N$.

2. From $\mathcal{N} = 1$ to $\mathcal{N} = 4$ via the Penrose limit

In this section we show that the Penrose limit of $AdS_5 \times M^5$ is the same as for $AdS_5 \times S^5$ (that is, (1.1)) provided that M^5 is a Sasaki-Einstein 5-manifold with a certain asymptotic behavior for the Kähler potential. We apply this to $AdS_5 \times \mathbb{T}^{11}$.

An $AdS_5 \times M^5$ compactification of type-IIB string theory, with smooth M^5, preserves some supersymmetry if and only if M^5 is a Sasaki-Einstein 5-manifold [9, 10]. This is equivalent to saying that it is the base of a 6-dimensional cone M^6 with metric

$$ds^2_{M^6} = dr^2 + r^2 d\Omega^2_{M^5_{S-E}}, \tag{2.1}$$

where M^6 is Kähler and Ricci-flat, in other words a Calabi-Yau space.

In turn, a Sasaki-Einstein 5-manifold M^5_{S-E} is a U(1) fibration over a 4-dimensional Kähler-Einstein space M^4_{K-E}. Under the assumption that M^5_{S-E} is smooth, its metric can be written as follows [15]:

$$ds^2_{M^5_{S-E}} = \left(d\beta + \frac{i}{2}(K_{,i}\, dz^i - K_{,\bar{i}}\, d\bar{z}^i)\right)^2 + K_{,i\bar{j}}\; dz^i d\bar{z}^j\,, \tag{2.2}$$

where $K(z, \bar{z})$ is the Kähler potential of M^4_{K-E}.

We introduce a constant R and scale the coordinates $z^i, \bar{z}^i$ so that the Kähler potential K depends only on $(z^i/R, \bar{z}^i/R)$. For the Penrose limit, we are interested in the behavior of the metric when $R \to \infty$. Requiring that in this limit the Kähler potential goes like

$$K(z, \bar{z}) \longrightarrow \frac{z\bar{z}}{R^2}, \tag{2.3}$$

one finds that

$$\frac{i}{2}(K_{,i}\, dz^i - K_{,\bar{i}}\, d\bar{z}^i) \longrightarrow \frac{i}{2R^2}(\bar{z}^i dz^i - z^i d\bar{z}^i)\,. \tag{2.4}$$

Consider now the full metric of $AdS_5 \times M^5_{S-E}$

$$\frac{ds^2}{R^2} = -dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho\, d\Omega_3^2 + \left(d\beta + \frac{i}{2}(K_{,i}\, dz^i - K_{,\bar{i}}\, d\bar{z}^i)\right)^2 + K_{,i\bar{j}}\, dz^i d\bar{z}^j\,.$$

With the geometry defined in these terms, the Penrose limit is obtained by scaling the AdS coordinate $\rho = r/R$ and then simply taking $R \to \infty$, dropping all terms that vanish in this limit. The metric above then becomes

$$ds^2 = -R^2 dt^2 + dr^2 + r^2 d\Omega_3^2 + R^2 d\beta^2 + i d\beta(\bar{z}^i dz^i - z^i d\bar{z}^i) + dz^i d\bar{z}^i\,. \tag{2.5}$$

Replacing

$$z^i \longrightarrow e^{i\beta} z^i \,, \tag{2.6}$$

and defining

$$x^+ = \frac{1}{2}(t+\beta)\,, \qquad x^- = \frac{1}{2}R^2(t-\beta)\,, \tag{2.7}$$

the metric is brought to the form

$$ds^2 = -4dx^+dx^- - (\vec{r}^{\,2} + z^i\bar{z}^i)(dx^+)^2 + d\vec{r}^{\,2} + dz^i d\bar{z}^i \,, \tag{2.8}$$

which is identical to eq. (1.1).

In the remainder of the paper we shall mainly focus on a special case of a Sasaki-Einstein 5-manifold: $\mathbb{T}^{1,1}$, the base of the conifold, whose metric can be written as [12, 16]

$$ds^2_{\mathbb{T}^{1,1}} = \frac{1}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 + \frac{1}{6}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2\theta_2 d\phi_2^2)\,. \tag{2.9}$$

This metric can be written in the general Sasaki-Einstein form eq. (2.2) by choosing the Kähler potential of the 4-dimensional base $P^1 \times P^1$ to be

$$K(z^i, \bar{z}^i) = \frac{2}{3}\sum_{i=1}^{2} \ln\left(1 + \frac{3}{2}\frac{z^i\bar{z}^i}{R^2}\right). \tag{2.10}$$

Since eq. (2.3) holds for this choice of the Kähler potential it follows from the discussion above that the Penrose limit of $AdS_5 \times \mathbb{T}^{1,1}$ is the same as the Penrose limit of $AdS_5 \times S^5$. Recalling that the original compactification breaks 3/4 of the supersymmetries of type-IIB, we see that in the Penrose limit this supersymmetry breaking becomes invisible, and maximal supersymmetry is restored. As we will see shortly, this has interesting consequences for the relationship between the field theory and its dual string.

In light of the enhancement of supersymmetry in the Penrose limit one might suspect that SUSY is not required all. Starting with a general non-supersymmetric $AdS_5 \times M^5$ and taking the Penrose limit, would one still end up with eq. (1.1)? To show that this is not the case, we consider $AdS_5 \times \mathbb{T}^{pq}$. The metric is [16, 17]

$$ds^2 = a^2(d\psi + p\cos\theta_1 d\phi_1 + q\cos\theta_2 d\phi_2)^2 + b^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + c^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2)\,,$$

where a, b and c are determined by p, q and the AdS_5 radius of curvature. The precise relation between a, b, c and p, q and R is not important for us and can be found, for example, in [17]. ψ has periodicity 4π and ϕ_1, ϕ_2 have periodicity 2π.

To take the Penrose limit it is useful to find a convenient null geodesic. This can be done by defining

$$\tilde{\psi} = \psi + p\phi_1 + q\phi_2\,, \qquad x^+ = \frac{1}{2}\left(t + a\tilde{\psi}\right), \qquad x^- = \frac{R^2}{2}\left(t - a\tilde{\psi}\right). \tag{2.11}$$

We expand for small θ_1 and θ_2:

$$b\theta_1 = \frac{r_1}{R}\,, \qquad c\theta_2 = \frac{r_2}{R}\,, \tag{2.12}$$

and take the $R \to \infty$ limit. After shifting the angular coordinates,

$$\varphi_1 = \phi_1 - \frac{ap}{2b^2}\left(x^+ - \frac{x^-}{R^2}\right), \qquad \varphi_2 = \phi_2 - \frac{aq}{2c^2}\left(x^+ - \frac{x^-}{R^2}\right), \tag{2.13}$$

JHEP03(2002)048

we find the following pp wave metric:

$$\begin{aligned} ds^2 \, &= -4dx^+dx^- + \sum_{i=1}^{4} dx_i^2 + \sum_{i=1}^{2}(dr_i^2 + r_i^2 d\varphi_i^2) - \\ &\quad - (dx^+)^2 \left(\sum_{i=1}^{4} x_i^2 + \frac{a^2p^2}{4b^4} r_1^2 + \frac{a^2q^2}{4c^4} r_2^2 \right) . \end{aligned} \tag{2.14}$$

For $p = q = 1$, we have $a = 1/3$, $b^2 = c^2 = 1/6$, so that we recover the metric (2.8).

For $p \neq q$, not all the bosons have the same mass in the light-cone gauge, but the solution still has the general pp wave form discussed in [5, 6]. Thus, it preserves at least 16 supersymmetries.[3] The tachyons present in the $AdS_5 \times \mathbb{T}^{pq}$ compactification [17] did not survive the pp wave limit, in agreement with the supersymmetry of the solution. Since the fermion masses in the light-cone gauge are determined solely by the 5-form, they are the same as in the supersymmetric case. Therefore, some of the fermion masses do not match bosons and the pp wave is not maximally supersymmetric [5, 6].

3. Field theory description of enhanced SUSY

In the previous section we saw that the Penrose limit of $AdS_5 \times \mathbb{T}^{11}$ is identical to the Penrose limit of $AdS_5 \times S^5$. The AdS/CFT duality implies that this should have non-trivial consequences for the dual field theories.

As was explained beautifully by BMN, from the dual field theory point of view taking the Penrose limit means focusing on a particular sector of single-trace operators of the $\mathcal{N} = 4$ SYM theory while taking the 't Hooft coupling to infinity. To be more precise we need to focus on "almost BPS" operators with large R-charge, J, which scales like the square-root of the 't Hooft coupling:

$$\lambda = g_{\rm YM}^2 N \longrightarrow \infty \,, \qquad \frac{J^2}{\lambda} = \text{finite} \,, \qquad \Delta - J = \text{finite} \,. \tag{3.1}$$

These operators are the ones relevant for describing strings propagating in the pp wave background. The demand that $\Delta - J$ remains finite follows from keeping the light-cone hamiltonian finite. For example, the light-cone vacuum is identified with $\operatorname{Tr} Z^J$, where $Z = \phi_1 + i\phi_2$ carries R-charge 1 (ϕ_a are the 6 adjoint scalars fields of the $\mathcal{N} = 4$ SYM theory). For a given J this is the unique state with $\Delta - J = 0$. The 8 transverse oscillations of the string correspond to inserting ϕ_a, $a = 3, 4, 5, 6$ and $D_k Z$, $k = 1, 2, 3, 4$, into the trace. The $\mathcal{N} = 4$ SYM theory also has 4 doublets of adjoint Weyl fermions; the 8 fermionic oscillators correspond to inserting them into the trace. It is important that the above exhausts the list of operators whose single insertion into the trace produces an operator with $\Delta - J = 1$. By studying their multiple insertions, BMN argued that the resulting single-trace operators are in one-to-one correspondence with the single string spectrum in the pp wave background (1.1).

[3] We are grateful to Nakwoo Kim, Neil Constable and Jose Figueroa-O'Farrill for pointing out an erroneous statement about supersymmetry for $p \neq q$ in an earlier version of this paper.

The result of the previous section implies that, even though the field theory dual to string theory on $AdS_5 \times S^5$ is not the same as the field theory dual to $AdS_5 \times \mathbb{T}^{11}$, that particular sector must be equivalent. The aim of this section is to explain how this comes about from the point of view of the field theory dual to $AdS_5 \times \mathbb{T}^{11}$ which has only $\mathcal{N} = 1$ supersymmetry. The $\mathcal{N} = 1$ superconformal field theory on the worldvolume of N D3-branes at a conifold singularity was first constructed in ref. [12] and has been extensively studied in subsequent works. We briefly review the essential features of this theory here. For details, the reader is referred to ref. [12].

The theory has a gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$, along with bi-fundamental superfields A_1, A_2 and B_1, B_2. The superfields are doublets of the first and second factors, respectively, of a global $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry. There is a $\mathrm{U}(1)_R$-symmetry under which the chiral multiplet all have charge $+1/2$, and a quartic superpotential

$$W = \lambda \epsilon^{ik} \epsilon^{jl} A_i B_j A_k B_l \,. \tag{3.2}$$

This superpotential is marginal at the conformal fixed point by virtue of the fact that the A_i, B_i acquire anomalous dimensions. Thus, in the CFT each of these fundamental fields has dimension $\Delta = 3/4$.

The first step toward finding the relevant sector in the theory which is dual to the pp wave is to determine the light-cone hamiltonian, $i\partial_{x^+}$, in terms of the field theory generators[4]

$$\Delta = i\partial_t \,, \qquad J = -2i\partial_\psi \,, \qquad J_3 = i\partial_{\phi_1} \,, \qquad J_3' = i\partial_{\phi_2} \,. \tag{3.3}$$

Here J is the $\mathrm{U}(1)_R$ charge, and J_3, J_3' are the diagonal generators of the two factors in the global symmetry group $\mathrm{SU}(2) \times \mathrm{SU}(2)$. We write

$$\partial_{x^+} = \frac{\partial t}{\partial x^+}\partial_t + \frac{\partial \psi}{\partial x^+}\partial_\psi + \frac{\partial \phi_i}{\partial x^+}\partial_{\phi_i} \,. \tag{3.4}$$

Using the relation between the original $\mathbb{T}^{1,1}$ coordinates in (2.9) and the coordinates where the metric assumes the pp wave form (2.14),

$$\begin{aligned} t &= x^+ + \frac{x^-}{R^2} \,, \qquad \psi = x^+ - \frac{x^-}{R^2} - \varphi_1 - \varphi_2 \,, \\ \phi_i &= \varphi_i + x^+ - \frac{x^-}{R^2} \,, \end{aligned} \tag{3.5}$$

we find the light-cone hamiltonian given in (1.5), i.e.

$$2P^- = H = \Delta - \frac{1}{2}J + J_3 + J_3' \,. \tag{3.6}$$

Analogously, we find

$$2P^+ = \frac{1}{R^2}\left(\Delta + \frac{1}{2}J - J_3 - J_3'\right) . \tag{3.7}$$

[4]The factor of 2 in J is due to the periodicity of ψ being 4π.

JHEP03(2002)048

	Δ	J	J_3	J'_3	H
A_1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1
A_2	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
B_1	$\frac{3}{4}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1
B_2	$\frac{3}{4}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
χ_{A_1}	$\frac{5}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	2
χ_{A_2}	$\frac{5}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	1
χ_{B_1}	$\frac{5}{4}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	2
χ_{B_2}	$\frac{5}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	1
ψ	$\frac{3}{2}$	1	0	0	1
$\tilde{\psi}$	$\frac{3}{2}$	1	0	0	1

Table 1: Dimensions and charges for chiral fields and gauginos.

	Δ	J	J_3	J'_3	H
$\overline{A}_1$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\overline{A}_2$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{3}{2}$
$\overline{B}_1$	$\frac{3}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
$\overline{B}_2$	$\frac{3}{4}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$
$\overline{\chi}_{A_1}$	$\frac{5}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\overline{\chi}_{A_2}$	$\frac{5}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{3}{2}$
$\overline{\chi}_{B_1}$	$\frac{5}{4}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
$\overline{\chi}_{B_2}$	$\frac{5}{4}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$
$\overline{\psi}$	$\frac{3}{2}$	-1	0	0	2
$\overline{\tilde{\psi}}$	$\frac{3}{2}$	-1	0	0	2

Table 2: Dimensions and charges for complex conjugate fields.

In tables 1 and 2, we make a list of the H values of the various fundamental fields in the SCFT. Then we will construct some gauge-invariant operators and explain how they are to be identified with the theory of strings on a pp wave background. In table 1, A_i, B_i refer to the scalar components of the chiral superfields described above, and χ_{A_i}, χ_{B_i} are their fermionic partners. ψ and $\tilde{\psi}$ are the gauginos of the two gauge groups. table 2 has the dimensions and charges of the complex conjugate fields.

From the tables, we see that the unique operator that should be identified with the light-cone vacuum is $\mathrm{Tr}(A_2B_2)^J$ which has $H=0$. These operators are analogous to $\mathrm{Tr}\, Z^J$ in the maximally supersymmetric theory of ref. [4]. Next, we turn to operators with $H=1$. Let us first consider the case $J=1$. The large-J case will be discussed shortly. From the table we find the following bosonic chiral operators: $\mathrm{Tr}\, A_1B_2, \mathrm{Tr}\, A_2B_1, \partial_k(\mathrm{Tr}\, A_2B_2)$, $k=1,2,3,4$. These are 6 of the necessary operators, which means that we are missing two bosonic operators.

For the fermionic operators, we find: $\mathrm{Tr}\, \chi_{A_2}B_2, \mathrm{Tr}\, A_2\chi_{B_2}$ with $J=1$. As the χ are 2-component Weyl fermions, these make 4 operators altogether. However, only two of them are the superpartners of a bosonic operator, namely the combination $\mathrm{Tr}(\chi_{A_2}B_2 + A_2\chi_{B_2})$. This is the variation of $\mathrm{Tr}\, A_2B_2$.

So far, we have found 6 chiral bosons and 2 chiral fermions with $H=1$. However, for the correspondence with the universal pp wave to work, we must find 2 additional bosons and 6 additional fermions. It turns out that there are precisely two non-chiral bosonic operators with $J=0$ that have $H=1$. The fact that the operators are non chiral should not come as a surprise. This is also the case with the $\mathcal{N}=4$ SYM theory when viewed as a $\mathcal{N}=1$ theory with three chiral superfields [18]. Consider, for example, the operators

$$Z^J V\,, \qquad Z^J \overline{V}\,, \tag{3.8}$$

where $Z = \phi^1 + i\phi^2$ and $V = \phi^3 + i\phi^4$. From the $\mathcal{N} = 4$ point of view both are chiral with protected dimensions. However only the first one is chiral in the $\mathcal{N} = 1$ language.

Indeed, much like in (3.8) the two extra operators with $H = 1$ involve the complex conjugate components of the scalars, and can be written as

$$\mathrm{Tr}\, A_2 \overline{A}_1\,, \qquad \mathrm{Tr}\, B_2 \overline{B}_1\,. \tag{3.9}$$

Unlike in (3.8) we do not have $\mathcal{N} = 4$ supersymmetry at our disposal to fix their dimension. The total dimension of these operators is not the naive sum of the dimensions of the constituents. In general we would not know how to compute their dimensions, but it turns out [17, 19] that they belong to the same supermultiplet as the currents generating the global $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry. Therefore, their dimension is equal to its free-field value, namely $\Delta = 2$.[5] Using the assignment of R-charge and the other global charges for the conjugate fields, which can be found in table 2, we see that indeed these operators have $H = 1$.

Let us turn now to the fermionic operators. The bosonic operators we described above have fermionic counterparts, whose dimension is correspondingly protected. These are $\mathrm{Tr}\, \overline{\chi}_{A_1} A_2$ and $\mathrm{Tr}\, \overline{\chi}_{B_1} B_2$, which provide us 4 additional fermionic operators. So we are still missing 2 more fermionic operators to complete the set of 8. These can be constructed by making use of the gauginos. Acccording to the analysis of ref. [19], the operators $\mathrm{Tr}\left(\psi(A_2B_2)^J + \tilde{\psi}(B_2A_2)^J\right)$ lie in short multiplets. Moreover, from table 1, we see that they have $H = 1$. Just as for the fermions in the chiral multiplets, here also we keep only the symmetric combination (under the interchange of the two gauge group factors) which is a protected operator. Thus, we have found the last 2 fermionic operators of $H = 1$, making up the collection of 8 operators that we propose to identify with the fermionic oscillators of the light-cone superstring in the pp wave background.

In light of this counting it is very natural to propose that the following 8 bosonic and 8 fermionic operators,

$$\begin{aligned}
\text{Bosonic}:\ & \mathrm{Tr}\, A_1B_2(A_2B_2)^J\,, \qquad \mathrm{Tr}\, A_2B_1(A_2B_2)^J\,,\\
& \mathrm{Tr}\, A_2\overline{A}_1(A_2B_2)^J\,, \qquad \mathrm{Tr}\, B_2\overline{B}_1(B_2A_2)^J\,,\\
& \partial_i\, \mathrm{Tr}(A_2B_2)^J\,,\\
\text{Fermionic}:\ & \mathrm{Tr}(\chi_{A_2}B_2 + A_2\chi_{B_2})(A_2B_2)^J\,,\\
& \mathrm{Tr}\, \overline{\chi}_{A_1}A_2(B_2A_2)^J\,, \qquad \mathrm{Tr}\, \overline{\chi}_{B_1}B_2(A_2B_2)^J\,,\\
& \mathrm{Tr}\left(\psi(A_2B_2)^J + \tilde{\psi}(B_2A_2)^J\right).
\end{aligned} \tag{3.10}$$

are the relevant ones to describe the BMN sector. To establish this one needs to show that these operators have $H = 1$ for all J. This follows immediately for the chiral operators.

[5] The argument goes as follows. The global symmetry requires that there is an SU(2) triplet of conserved currents of dimension 3. The $J_3 = -1$ component of this triplet is $A_2\partial_\mu\overline{A}_1 - (\partial_\mu A_2)\overline{A}_1 +$ fermions. The scalar $A_2\overline{A}_1$ is related by the supersymmetry generators to this current and hence has dimension 2.

However, as far as we can see, there is no field theory argument that protects the non-chiral operators. That is, the argument used above for operators (3.9) cannot be generalized to $J > 0$. Since the Penrose limit involves sending the 't Hooft coupling to infinity, it is sufficient for us to show that the operator of the form

$$O_k = \mathrm{Tr}\, A_2 \overline{A}_1 (A_2 B_2)^k\,, \tag{3.11}$$

has $H = 1$ in the region where the SUGRA approximation is valid. The quantum numbers of O_k are $J = k$, $\mathbf{J} = 1 + (k/2)$, $\mathbf{J}' = k/2$ (by $\mathbf{J}$ we denote the angular momentum of SU(2) and analogously for the SU(2)$'$). According to the AdS/CFT correspondence,

$$\Delta = 2 + \sqrt{4 + (mL)^2}\,, \qquad (mL)^2 = \lambda + 16 - 8\sqrt{\lambda + 4}\,, \tag{3.12}$$

where [17]

$$\lambda = 6\left[\mathbf{J}(\mathbf{J}+1) + \mathbf{J}'(\mathbf{J}'+1) - J^2/8\right]\,, \tag{3.13}$$

is the value of the laplacian on $\mathbb{T}^{1,1}$. For the operator O_k we find

$$(mL)^2 = \left(\frac{3k}{2}\right)^2 - 4\,, \tag{3.14}$$

so that $\Delta = 2 + 3k/2$, which in turn implies $H = 1$.

The fact that there is no apparent field theory argument to protect the dimension of these operators suggests that there are non-trivial α' corrections away from the supergravity approximation. This fits with the fact that the symmetry should be enhanced only in the strict Penrose limit.

4. $S^5/\mathbb{Z}_2$ theory: operator spectrum and strings on orbifolded pp waves

The compactification of type-IIB string theory on $AdS_5 \times S^5/\mathbb{Z}_2$ is an example where the compact 5-manifold is not smooth, but has a singular S^1 submanifold [20]. This compactification is dual to the theory on D3-branes transverse to a $\mathbb{Z}_2$ ALE singularity, and the world-volume theory is an $\mathcal{N} = 2$ SCFT.

The pp wave limit of this geometry can be obtained very simply and one finds that the metric is the universal one, as given in eq. (2.8) above. However, the coordinates z_a, $i = 1, 2$, are identified under the $\mathbb{Z}_2$ action $z^i \to -z^i$, and so there is an ALE singularity that survives in the transverse space. We believe, however, that in the pp wave case there is little physical difference between the untwisted and twisted sectors. This is because the untwisted states have have no continuous transverse momenta; they are essentially localized near the orbifold plane by the metric so that transverse excitations have gaps. The same is true for the twisted sector states. We will see that the gauge theory operators dual to the untwisted and twisted states are also very similar.

Although the field content of the $\mathcal{N} = 2$ $\mathbb{Z}_2$ orbifold gauge theory is similar to that for $AdS_5 \times \mathbb{T}^{1,1}$ (and the two field theories are related by a massive perturbation, as argued in ref. [12]), the theory is closer to the maximally supersymmetric one in an important aspect: the fundamental fields have no anomalous dimensions, so we have scalars and fermions of

JHEP03(2002)048

canonical dimension $\Delta = 1, 3/2$, respectively. The R-symmetry group is $\mathrm{SU}(2) \times \mathrm{U}(1)$. The gauge group is $\mathrm{SU}(N) \times \mathrm{SU}(N)$, and each gauge multiplet contains a complex adjoint scalar, which we denote $\phi, \tilde{\phi}$. In addition there are bi-fundamental fields which can be represented as $\mathcal{N} = 1$ chiral multiplets A_i, B_i, though in $\mathcal{N} = 2$ they of course combine into hypermultiplets.

In this theory the adjoint scalars $\phi, \tilde{\phi}$ are associated with positions of the (fractional) D3-branes within the orbifold fixed sixplane. We choose the $\mathrm{U}(1)$ subgroup of R-symmetry to act only on these fields, but not on the hypermultiplets. Hence we can form gauge-invariant operators $\mathrm{Tr}\,\phi^J$ and $\mathrm{Tr}\,\tilde{\phi}^J$ which have $\Delta - J = 0$. Each of these appears to be independently analogous to the operators $\mathrm{Tr}\,Z^J$ of ref. [4], which at large J are identified with the vacuum state of the string. Thus we seem to have two distinct candidates for the string vacuum. However, note that these two operators are exchanged by the $\mathbb{Z}_2$ that exchanges the two gauge group factors. We associate the symmetric combination $\mathrm{Tr}(\phi^J + \tilde{\phi}^J)$ with the ground state in the untwisted sector of the string, while the antisymmetric combination $\mathrm{Tr}(\phi^J - \tilde{\phi}^J)$ is associated with the ground state in the twisted sector localized at the singularity. More precisely, the operator $\mathrm{Tr}(\phi^J + \tilde{\phi}^J)$ corresponds to a graviton moving with longitudinal momentum J while the operator $\mathrm{Tr}(\phi^J - \tilde{\phi}^J)$ describes a member of the six-dimensional tensor multiplet moving with longitudinal momentum J. More generally, we define the operator P which interchanges the two gauge groups, and interchanges ϕ with $\tilde{\phi}$, and A_i with B_i, $i = 1, 2$. Then, given any operator O, $O + PO$ and $O - PO$ belong to the untwisted and twisted sectors, respectively.

The operators $\mathrm{Tr}(\phi^J \pm \tilde{\phi}^J)$ are rotated by the $\mathrm{U}(1)$ factor in the R-symmetry group $\mathrm{SU}(2) \times \mathrm{U}(1)$. The R-symmetry does not act on the A_i, B_j, so each of these fields has $\Delta - J = 1$. The new feature, with respect to the maximally supersymmetric case, is that we cannot form gauge-invariant operators out of A_i or B_j alone since they are bi-fundamentals. Nevertheless, we can write down the following set of 'excited' operators where these operators appear in pairs:

$$\begin{aligned}
O^n_{ij} &= \sum_{l=0}^{J} e^{\pi i n l/J}\, \mathrm{Tr}(\phi^l A_i \tilde{\phi}^{J-l} B_j)\,, \\
\overline{O}^n_{ij} &= \sum_{l=0}^{J} e^{\pi i n l/J}\, \mathrm{Tr}(\phi^l \overline{B}_i \tilde{\phi}^{J-l} \overline{A}_j)\,, \\
O^n_{Aij} &= \sum_{l=0}^{J} e^{\pi i n l/J}\, \mathrm{Tr}(\phi^l A_i \tilde{\phi}^{J-l} \overline{A}_j)\,, \\
O^n_{Bij} &= \sum_{l=0}^{J} e^{\pi i n l/J}\, \mathrm{Tr}(\phi^l \overline{B}_i \tilde{\phi}^{J-l} B_j)\,.
\end{aligned} \tag{4.1}$$

We may form their untwisted and twisted combinations, which are, respectively, symmetric and antisymmetric under the $\mathbb{Z}_2$ symmetry P interchanging the two gauge groups. The resulting operators correspond to $a^i_{-n/2} \tilde{a}^j_{-n/2}$ acting on the (un)twisted light-cone vacuum, i.e. they describe oscillations of the closed string in the z^i directions, which we recall were orbifolded by a $\mathbb{Z}_2$ action. Note that for odd n they have antiperiodic boundary conditions.

In the gauge theory operators we indeed pick up (-1) by moving A_i or B_j around the loop: for example, the $\mathrm{Tr}(\phi^J A_i B_j)$ and $\mathrm{Tr}(\tilde{\phi}^J B_j A_i)$ terms in O^n_{ij} have a relative minus sign.[6] For $n = 0$ they should correspond to the massless string states, hence their dimensions should be protected so that these operators have $\Delta = J + 2$. It may seem peculiar that non-chiral operators such as $\overline{O}^n_{ij}$ should be protected, but we already noted a similar phenomenon in the $\mathcal{N} = 4$ case: many of the operators that are chiral from the $\mathcal{N} = 4$ point of view do not look chiral when written in terms of $\mathcal{N} = 1$ superfields [18].

Discussion of the oscillations of the string along the $\vec{r}$ directions, which arise from AdS_5, proceeds by analogy with [4]. Since the orbifold group does not act on these directions, the corresponding oscillators do not have to appear in pairs, and there cannot be twisted boundary conditions. Each such insertion should have unit light-cone hamiltonian as before, and to these we associate the four $\Delta = J + 1$ operators $\partial_k \,\mathrm{Tr}(\phi^J \pm \tilde{\phi}^J)$, $k = 1, 2, 3, 4$, where the $\pm$ signs are chosen for the untwisted and twisted sectors, respectively. The superpartners of these excitations are the operators

$$\mathrm{Tr}(\psi\phi^J) \pm \mathrm{Tr}(\tilde{\psi}\tilde{\phi}^J)\,. \tag{4.2}$$

where ψ $(\tilde{\psi})$ is one of the four complex fermions from the $\mathcal{N} = 2$ vector multiplet which are the adjoints of the first (second) $\mathrm{SU}(N)$.

The discussion above, although somewhat sketchy, shows that it is indeed possible to construct all the string states, in a $\mathbb{Z}_2$ orbifold of the pp wave by a reflection of 4 transverse coordinates, out of the gauge invariant operators of the $\mathcal{N} = 2$ superconformal $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory.

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JHEP03(2002)048

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